CAP High School Prize Exam

11 April 2003
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: ______________________ Given Name: ______________

Home Address: ____

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______________________ Postal Code: ______________

Telephone: (   ) ________________ E-mail: ______________________

School: ______________________________ Grade: ______________

Physics Teacher: ______________________________

Date of Birth: ______________________________ Sex: ______________

Citizenship: ______________________

For how many years have you studied in a Canadian school? ____

Would you prefer further correspondence in French or English? ____

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Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2003 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Radius of Earth	$R_E = 6.38 \times 10^3$ km
Mass of Earth	$M_E = 6.0 \times 10^{24}$ kg
Mass of Sun	$M_S = 2.0 \times 10^{30}$ kg
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^8$ km
Acceleration due to gravity	$g = 9.80\,\mathrm{m/s^2}$
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C²
Speed of sound in air	$v_s = 343\,\mathrm{m/s}$
Energy conversion	1 eV = 1.6×10^{-19} J

Part A: Multiple Choice

Question 1

A parallel-plate capacitor holds charge q and is not connected to anything. The distance between the plates is now increased. The electrical energy stored on the capacitor

(a) decreases;
(b) remains the same;
(c) increases;
(d) can do any of the above, depending on how the capacitance changes.

Question 2

When a mechanical or electromagnetic wave goes from one medium to another, it undergoes a change in

(a) amplitude only; (b) both speed and wavelength;
(c) speed only; (d) wavelength only.

Question 3

Two identical rooms in a perfectly insulated house are connected by an open doorway. The temperature in the two rooms are maintained at different values. The room which contains more air molecules is

(a) the one with the higher temperature;
(b) the one with the lower temperature;
(c) the one with the higher pressure;
(d) neither, since both have the same volume.

Question 4

Three airplanes, A, B and C, each release an object from the same altitude and with the same initial speed v_0 with respect to the ground.. At the moment their object is released, A is flying horizontally, B is flying upward at an angle θ with respect to the horizontal, and C is flying at the same angle θ as B but downward with respect to the horizontal. Assuming the ground to be horizontal and neglecting any aerodynamical effect, the speeds v at which the three objects will hit the ground satisfy

(a) $v_A = v_B < v_C$; (b) $v_A > v_B = v_C$;
(c) $v_A < v_B < v_C$; (d) $v_A = v_B = v_C$.

Question 5

Two identical conducting spheres, A and B, carry equal electric charge. They are separated by a distance much larger than their diameter and exert an electrostatic force F on each other. A third identical conducting sphere C is initially uncharged and far away from A and B. Sphere C is then brought briefly into contact with sphere A, then with sphere B, and finally removed far away. The electrostatic force between A and B is now

(a) $3F/8$; (b) $F/2$;
(c) $F/4$; (d) $F/16$.

Question 6

On the ground, the Earth exerts a force F_0 on an astronaut. The force that the Earth exerts on this astronaut inside the Space Shuttle in low Earth orbit, 300 km above the ground, is

(a) a little less than F_0;
(b) a little more than F_0;
(c) exactly F_0;
(d) zero, since the astronaut is weightless when in orbit.

Question 7

A person is swinging a ball at the end of a string of length ℓ with constant speed v. The work done by the tension T in the string over one revolution is

(a) 0; (b) $mv^2/2$;
(c) $2\pi\ell T$; (d) undetermined by the information given.

Question 8

The pressure exerted by a gas on the walls of the vessel that contains it is due to the

(a) change in kinetic energy of the gas molecules as they strike the walls;

(b) collisions between the gas molecules;

(c) repulsive force between the gas molecules;

(d) change in momentum of the gas molecules as they strike the walls.

Question 9

A Martian creature similar to an Earth frog jumps with an initial speed v_0 and attains the range R over horizontal ground. The maximum possible height reached by the creature, neglecting friction in the tenuous Martian air, is (θ being the launch angle):

(a) $\frac{R}{4}\tan\theta$;

(b) $\frac{R}{4}\sin\theta$;

(c) $\frac{R}{2}\tan\theta$;

(d) undetermined because of missing data.

Question 10

The work done to accelerate a truck on a horizontal road from rest to speed v

(a) is less than that required to accelerate it from v to $2v$;

(b) is equal than that required to accelerate it from v to $2v$;

(c) is more than that required to accelerate it from v to $2v$;

(d) may be any one of the above since it depends on the force acting on the truck and the distance over which it acts.

Question 11

If the Earth did not rotate on its axis, the magnitude of the gravitational acceleration at the Equator would be about

(a) 0.003% larger; (b) 0.3% larger;

(c) 0.3% smaller; (d) 0.003% smaller.

Question 12

You want to apply a force on a box so that it moves with constant speed across a horizontal floor. The coefficient of kinetic friction between the box and the floor is μ_k. Of the four following cases, the force you apply on the box will be smallest when you

(a) push on it with a force applied at an angle $0 < \theta < 90°$ downward from the horizontal;

(b) pull on it with a force applied at the same angle θ as in (a), upward from the horizontal;

(c) do either (a) or (b) since the applied force is the same;

(d) push or pull with a force applied horizontally.

Question 13

An electric current runs counterclockwise in a rectangular loop around the outside edge of this page, which lies flat on your table. A uniform magnetic field is then turned on, directed parallel to the page from top to bottom. The magnetic force on the page will cause

(a) the left edge to lift up; (b) the right edge to lift up;

(c) the top edge to lift up; (d) the bottom edge to lift up.

Question 14

A car has a maximum acceleration of $3.0\,\mathrm{m/s^2}$. Its maximum acceleration while towing another car twice its mass, assuming no skidding, would be

(a) $3.0\,\mathrm{m/s^2}$; (b) $1.5\,\mathrm{m/s^2}$;

(c) $1.0\,\mathrm{m/s^2}$; (d) $0.5\,\mathrm{m/s^2}$.

Question 15

Two satellites of equal mass, A and B, are in concentric orbits around the Earth. The distance of B from Earth's centre is twice that of A. The ratio of the centripetal force acting on B to that acting on A is

(a) 1 ; (b) $\sqrt{1/2}$;

(c) 1/2 ; (d) 1/4 .

Question 16

A proton sits at coordinates $(x, y) = (0, 0)$, and an electron at (d, h), where $d \gg h$. At time $t = 0$, a uniform electric field $\boldsymbol{E}$ of unknown magnitude but pointing in the positive y direction is turned on. Assuming that d is large enough that the proton-electron interaction is negligible, the y coordinates of the two particles will be equal (at equal time)

(a) at about $y = d/2000$;

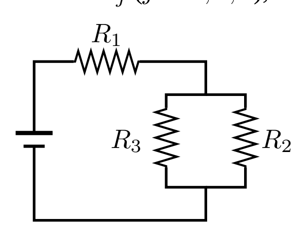
(b) at an undetermined value since E is unknown;

(c) at about $y = d/43$;

(d) nowhere: they move in opposite directions.

Question 17

In the circuit below we increase the resistance R_2. If I_j is the current through resistor R$_j$ ($j = 1, 2, 3$), then



(a) I_1 and I_2 both increase;

(b) I_1 decreases and I_2 increases;

(c) I_1 and I_2 both decrease;

(d) I_1 increases and I_2 decreases.

Question 18

Two carts, A and B, are placed on an air track. They are made of the same material and *look* identical. B is given a constant speed and collides elastically with A at rest. After the collision, both carts move in the same direction. One concludes that

(a) A is hollow;

(b) B is hollow;

(c) A and B are identical;

(d) any of the first three answers is possible.

Question 19

The smallest length scale known in physics is the Planck length. It is an important ingredient in some currrent cosmological theories. Which of the following expressions could represent this Planck length? (see Data table.)

(a) $\sqrt{e^2/hc}$; (b) $\sqrt{hc/G}$;

(c) $\sqrt{Ghc}$; (d) $\sqrt{hG/c^3}$.

Question 20

The Webb space telescope, scheduled to be launched in 2010, will have a mirror $6\,\text{m}$ in diameter. Compared with the Hubble space telescope, whose mirror has a $2.4\,\text{m}$ diameter, it will be able to resolve objects whose angular separation is about

(a) 2.5 times smaller;

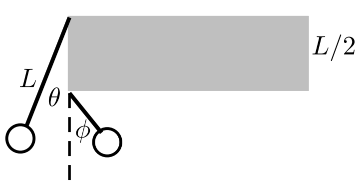
(b) 5 times smaller;

(c) an order of magnitude smaller;

(d) the same: the larger mirror only increases the amount of light gathered.

Question 21

A simple pendulum of length L is suspended from the top of a flat beam of thickness $L/2$. The bob is pulled away from the beam so that it makes an angle $\theta < 30°$ with the vertical, as shown in the figure. It is then released from rest. If ϕ is the maximum angular deflection to the right, then

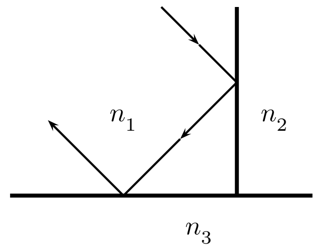


(a) $\phi = \theta$; (b) $\phi < \theta$;

(c) $\theta < \phi < 2\theta$; (d) $\phi \geq 2\theta$.

Question 22

In the diagram below, light is incident on the interface between media 1 and 2 at exactly the critical angle, and is totally reflected. The light is then also totally reflected at the interface between media 1 and 3, after which it travels in a direction opposite to its initial direction. The two interfaces are perpendicular. The media must have a refraction index n such that



(a) $n_1 < n_2 < n_3$; (b) $n_1^2 - n_3^2 \geq n_2^2$;

(c) $n_1^2 - n_2^2 \geq n_3^2$; (d) $n_1^2 + n_2^2 \geq n_3^2$.

Question 23

For the sake of science a physicist jumps attached to the end of a bungee cord, carrying sound measuring equipment. As he swings up and down vertically with a period of $6.0\,\text{s}$, he monitors the frequency of a sound source on the ground directly below him, and observes a difference of $84\,\text{Hz}$ between the maximum and minimum frequency of the source. If the source emits at a constant $1370\,\text{Hz}$, and assuming no significant attenuation of his oscillations over the duration of the measurements, the amplitude of his oscillations is closest to

(a) $10\,\text{m}$; (b) $20\,\text{m}$;

(c) $32\,\text{m}$; (d) $15\,\text{m}$.

Question 24

A person pulls a box along the ground at constant speed. Considering the Earth and the box together as a system, which of the following is true about the net force F exerted by the person on this system and the work W she does on it?

(a) $F = 0$ and $W = 0$; (b) $F \neq 0$ and $W = 0$;

(c) $F = 0$ and $W \neq 0$; (d) $F \neq 0$ and $W \neq 0$.

Question 25

A magnet moves inside a coil. Which of the following factors can affect the emf induced in the coil?

I. the speed at which the magnet moves

II. the strength of the magnet

III. the number of turns in the coil

(a) I only; (b) I and II only;

(c) II and III only; (d) I, II and III.

* * *

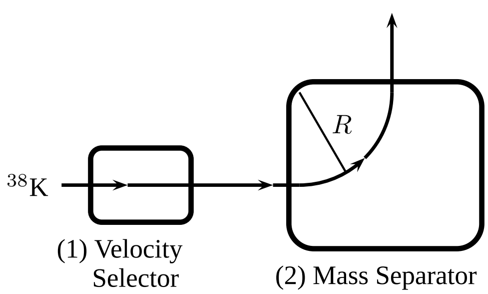
Part B

Problem 1

At TRIUMF, a large experimental particle and nuclear physics research facility on the campus of the University of British Columbia, one major programme involves the production of intense beams of unstable isotopes of alkali atoms (potassium K, rubidium Rb, francium Fr). These have the advantage that since their valence shell contains only one electron, their closed shell structure when they are ionised simplifies calculations.

Many isotopes are produced when bombarding a calcium oxide target with 0.5 GeV protons from the TRIUMF accelerator. Until recently, the desired isotope was selected by means of the TRIUMF Isotope Separator On-Line (TISOL)—now decommissioned and replaced by a combined separator/accelerator called ISAC—and sent as a low-speed beam to experimental areas.

You are asked to design a (much) simplified version of TISOL. More specifically, you want to select ^{38}K ions whose energy is 20 keV. ^{38}K has a mass of 6.3×10^{-26} kg. Separation should proceed in two steps, as illustrated below.



The figure shows the desired path of a 20 keV ^{38}K ion through the system. This path is to be achieved by means of suitable *uniform* time-independent electromagnetic fields. Interactions between ions can be neglected here.

(a) In the first step, out of all ions (^{38}K or not) entering the velocity selector from the left, only those that have a speed corresponding to a 20 keV ^{38}K ion should be undeflected. Suggest a field configuration that can do this, draw a sketch showing the direction of the field(s), and derive as much information as you can about the magnitude of the field(s).

(b) In the second step, only ^{38}K ions should be deflected so that the radius R of their trajectory is 2.1 m. Again, suggest a suitable field configuration for this, draw a sketch showing the direction of the field(s), and derive as much information as you can about the magnitude of the field(s).

Problem 2

Tides are mainly caused by the gradient (or variation) of the gravitational force of the Moon and of the Sun across the Earth's diameter. Large water masses, such as Earth's oceans, bulge along the direction of the gradient and are pinched in the perpendicular direction. As the Earth rotates, the bulges (high tide regions) move across the surface of the Earth.

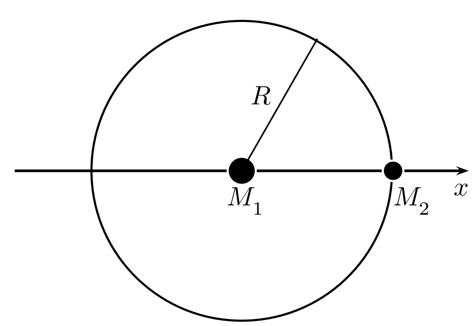
Now the Bay of Fundy, between New Brunswick and Nova Scotia, is reputed to have the highest tides in the world. Their amplitude is only about a metre at the mouth, or entrance, of the bay, whereas at the other end, 260 km away, the amplitude reaches up to 16 m. You may assume that the relevant speed of water waves in the bay (for *very long* wavelengths and shallow enough depth) is about 25 m/s. The bay is narrow compared to its length.

Using the above data, investigate whether the unusually high tides could be the result of a resonance excited by the Moon in the bay. Assume that the depth of the bay is uniform, and neglect the influence of the Sun. Hint: calculate the period of the water oscillations in the bay.

Problem 3

The next-generation large space telescope is scheduled for launch in 2010. It will be put in orbit around the Sun, in a special zone where its distance relative to the Earth and to the Sun can remain constant. The location of such zones was first calculated in the XVIII$^{\text{th}}$ century by the French-Italian mathematician Joseph-Louis Lagrange.

Even though it relies on approximations, Lagrange's full solution is fairly involved, but you should still be able to make a good qualitative guess at a partial solution. Consider two point masses, M_1 and M_2, referring respectively to the Sun and the Earth. Both orbit around their common centre of mass at angular velocity ω and with a period of one year. These orbits are *circular* to a good approximation, and the distance R between M_1 and M_2 is constant. Since $M_1 \gg M_2$, the motion of M_1 is not detectable at the scale of the figure below and can be neglected.



We wish to find where, on the line that joins M_1 and M_2, an object of mass m can sit so that it also orbits the centre of mass (which we can take to be at M_1's position) with the *same*, constant, angular velocity ω. We can also safely assume that m is so small that it does not influence the motion of M_1 and M_2.

(a) Write down the equation that must be satisfied by the forces acting on the orbiting m in terms of ω, M_1, M_2, R and x, where x is the distance between m and M_1.

(b) Show that in the limit $M_2 \ll M_1$, this condition can be written

$$u^3 - 1 \; = \; \pm \frac{\alpha\, u^2}{(1 \pm u)^2}$$

where $u \equiv x/R$, and $\alpha \equiv M_2/M_1$.

(c) Do not attempt to solve this algebraic equation. Instead, use physical arguments to find how many solutions there are for x and where roughly the Lagrange zones are positioned on the x axis. Provide a qualitative sketch based on the above figure, and explain your reasoning.

(d) The Webb space telescope (as it is called) will operate at a temperature of about 35 K and, therefore, it should be shielded from heat sources at all times, while having as unobstructed a view of the sky as possible. Discuss which (if any) of your solutions is most suitable for the Webb telescope.

(e) If there are other Lagrange zones off the x axis, they cannot be found from the equation in (b), but what would be their minimum number? Justify your answer.

Hint: To a good approximation Kepler's Third Law for the system is $\omega^2 R^3 \; = \; GM_1$.

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