Canadian Association of Physicists
1996 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273$°C

Part A: Multiple Choice

Question 1

A toy for firing a ball vertically consists of a vertical spring which is compressed by 0.10 m. A force of 10.0 N is needed to hold the spring at that compression. If a ball of mass 0.050 kg is placed on the compressed spring and the spring is released, the ball will reach a height (above its initial position) of:

(a) 1.0 m
(b) 1.2 m
(c) 1.4 m
(d) 1.6 m

Question 2

A rescue plane is flying horizontally with a speed of 30 m/s and at an altitude of 125 m above the sea when it drops a warning flare. Neglecting air resistance and assuming that the plane does not change its course, speed, or altitude, how far from the plane is the flare when it hits the water?

(a) 146 m
(b) 195 m
(c) 125 m
(d) 150 m

Question 3

A loudspeaker is placed over the open end of a pipe. By changing the frequency of the sound from the speaker, it is found that the pipe has resonances at 700 Hz and 900 Hz but not 800 Hz. This means that:

(a) The pipe is closed at one end and the fundamental is 100 Hz.
(b) The pipe is closed at one end and the fundamental is 200 Hz.
(c) The pipe is open at both ends and the fundamental is 100 Hz.
(d) The pipe is open at both ends and the fundamental is 200 Hz.

Question 4

In a television tube, electrons are accelerated from rest through a potential difference of 1600 V. What is the speed of the electrons after this acceleration?

(a) 16,000 km/s
(b) 20,000 km/s
(c) 24,000 km/s
(d) 28,000 km/s

Question 5

The electron-volt is a measure of:

(a) charge
(b) current
(c) electric field strength
(d) energy

Question 6

A current of 5 A passes along a wire of length 1.0 m. The wire is at right angles to a uniform magnetic field of strength 0.15 T. The force acting on the wire is:

(a) 0
(b) 0.75 N
(c) 33 N
(d) 0.03 N

Question 7

An 8 μF capacitor is charged to a potential of 120 V. How much work was required to do this?

(a) 2.7×10^{-12} J
(b) 1.2×10^{-1} J
(c) 9.6×10^{-4} J
(d) 5.8×10^{-2} J

Question 8

A flashlight is operated on four batteries placed in series. Each battery has an internal resistance r. If one of the batteries is accidentally placed the wrong way around, the total internal resistance of the four cells will now be:

(a) $r/4$
(b) $3r$
(c) $r/5$
(d) $4r$

Question 9

A ray of light is passing from air into glass. If the angle of incidence, with respect to the normal to the interface, is increased:

(a) Total internal reflection will occur when the angle of incidence equals the critical angle.
(b) Total internal reflection will occur when the angle of incidence is less than the critical angle.
(c) Total internal reflection will occur when the angle of incidence is greater than the critical angle.
(d) The refractive angle will increase but there will be no total internal reflection.

Question 10

A mass of 20 g is hung from the end of a light vertical spring and is set oscillating with an amplitude of 10 cm. Its total energy is found to be 4 J. If the mass is now replaced with a mass of 40 g and it is again set oscillating with an amplitude of 10 cm, its total energy is now:

(a) 2 J
(b) 4 J
(c) 5.6 J
(d) 8 J

Question 11

A 65 kg girl, riding in an elevator, weighs herself by standing on a scale. What is the reading of the scale if the elevator is accelerating downwards at 2 m/s^2?

(a) 65.0 kg
(b) 0 kg
(c) 51.7 kg
(d) 78.3 kg

Question 12

A highway curve of radius 30 m is banked so that a car travelling at 40 km/hr can travel around it without slipping even if there is no friction between the car's tires and the road surface. Without friction, a car travelling faster than this will slide up the curve, while a car travelling slower will slide down the curve. Find the angle of elevation of the banked highway curve.

(a) 67°.
(b) 23°.
(c) 45°.
(d) 90°.

Question 13

A ball is thrown upwards into the air rising to a height h. Accounting for the "real life" condition of air resistance, the time t_1 that the ball takes to rise to its maximum height and the time t_2 that it takes to fall back down to its initial position obey:

(a) $t_1 = t_2$
(b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 + t_2 = \sqrt{8h/g}$

Question 14

An airplane flies northwards from town A to town B and then back again. There is a steady wind blowing towards the north so that for the first stage of the trip, the airplane is flying in the same direction as the wind and for the return half of the journey, the airplane is flying directly into the wind. The total trip time T_w, as compared to the total trip time in the absence of any wind T_o, obeys:

(a) $T_w = T_o$
(b) $T_w > T_o$
(c) $T_w < T_o$
(d) $T_w = 2T_o$

Question 15

The space shuttle moves in a circular orbit about the earth at a constant speed. To change the orbit radius, the crew temporarily activates the main engine which accelerates the space shuttle in the direction of its motion. After the main engine is switched off again, the shuttle will be in an elliptical orbit with:

(a) a larger average radius and a lower average speed.
(b) a larger average radius and a higher average speed.
(c) a smaller average radius and a lower average speed.
(d) a smaller average radius and a higher average speed.

Question 16

Which of the following expressions has the correct units to represent the radius of a hydrogen atom in its ground state? (only one expression is correct)

(a) $\epsilon_o h^2/\pi m_e e^2$
(b) $h^2 m_e e^2/4\pi\epsilon_o$
(c) $m_e c^2/h^2 e^4$
(d) $e^4 m_e/8\epsilon_o^2 h^2$

Question 17

If the electric field is zero within some region of space, the electric potential within that region:

(a) must be zero.
(b) must be positive.
(c) must be negative.
(d) must be a constant value.

Question 18

In comparing the properties of visible light waves to microwaves, which of the following statements is FALSE?

(a) Visible light waves travel at the same speed in glass as do microwaves.
(b) Visible light waves have a higher frequency in glass than do microwaves.
(c) Visible light waves travel at the same speed in vacuum as do microwaves.
(d) Both visible light waves and microwaves can be refracted by glass.

Question 19

A boy sits at the top of a hemispherical mound of ice of radius r. He begins to slide (from rest) downwards without friction. At what height above the ground does he leave contact with the ice?

(a) The boy does not leave contact with the ice.
(b) $r/3$
(c) $r/2$
(d) $2r/3$

Question 20

A rocket for mining the asteroid belt is designed like a large scoop. It is approaching asteroids at a velocity of 10^4 m/s. The asteroids are much smaller than the rocket. If the rocket scoops asteroids at the rate of 100 kg/s, what thrust (force) must the rocket's engines provide in order for the rocket to maintain a constant velocity? Ignore any variation in the rocket's mass due to the burning of fuel.

(a) 10^3 N
(b) 10^6 N
(c) 10^9 N
(d) 10^{12} N

Question 21

The sensitivity of the human ear is greatest near 3000 Hz. This result is best explained by the fact that:

(a) the ear canal is a resonant cavity with a fundamental frequency near 3000 Hz.
(b) at a frequency of 3000 Hz the wavelength of sound is approximately the distance between the ears.
(c) 3000 Hz is the frequency at which the skull resonates.
(d) none of the above statements explains the result.

Question 22

A long piece of rubber is wider than it is thick. When it is stretched in length by some amount:

(a) its thickness decreases but its width increases.
(b) its thickness decreases but its width remains constant.
(c) its thickness increases but its width decreases.
(d) both its thickness and width decrease.

Question 23

A new medical procedure to cure near-sightedness involves changing the shape of the eye's lens using a precisely controlled laser beam. A near-sighted eye cannot focus clearly on distant objects. To improve the vision of the patient, we must:

(a) uniformly decrease the thickness of the lens.
(b) uniformly increase the thickness of the lens.
(c) make the middle of the lens thinner.
(d) make the outer rim of the lens thinner.

Question 24

A proton travels in a circular orbit of radius 1 cm in a magnetic field of strength 0.5 T. The kinetic energy of the proton is:

(a) 3.35×10^{-27} J
(b) 1.67×10^{-27} J
(c) 3.83×10^{-16} J
(d) 1.91×10^{-16} J

Question 25

A metal rod 30 cm long moves at 8 m/s in a plane perpendicular to a magnetic field of 0.05 T. The velocity of the rod is in a direction perpendicular to its length. The potential difference induced between the ends of the rod is:

(a) 48 V
(b) 6.40×10^{-20} V
(c) 120 mV
(d) A potential difference will not be induced in this case.

Part B

Question 1

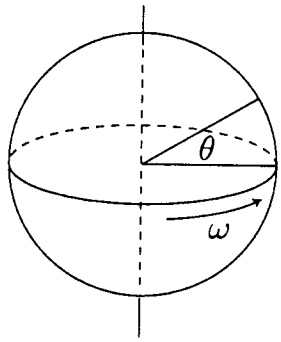
The study of the acceleration due to gravity of the earth can be very interesting. A variety of factors, such as the earth's spin and altitude, result in variations of "g". In this question, we shall investigate these variations.

(a) First let's assume that the earth is a non-spinning sphere. Find an expression for "g" at the earth's surface in terms of the earth's mass M_E, radius R_E and G, the gravitational constant.

(b) Modify your expression from part (a) to give the acceleration due to gravity at an altitude of h above the earth's surface. Simplify your expression in the limit that $h \ll R_E$ (the altitude is much less than the radius of the earth). Compare the weights of a 90 kg mountaineer at sea level and at the top of Everest at an altitude of about 8000 m. Hint: you may use the approximation that $1/(1+x)^2 \approx 1 - 2x$ if $\mid x \mid \ll 1$.

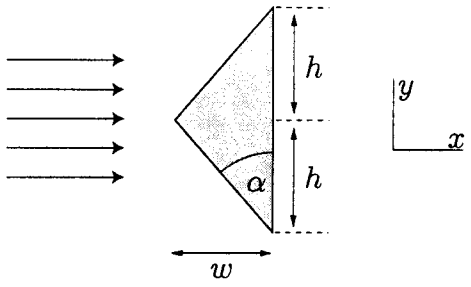
(c) The acceleration due to gravity depends on latitude because the earth is spinning. Find the percentage difference between g at the poles and at the equator due to the earth's spin. Hint: consider the weight of an object at the two locations.

(d) Find an expression for the magnitude of the freefall acceleration of an object near the surface of the earth as a function of the latitude θ. Assume that the earth is spherical and spinning.



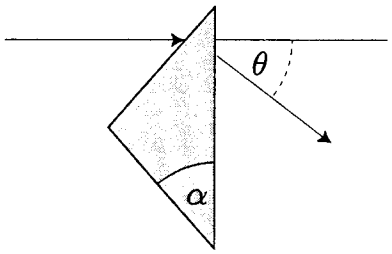
Question 2

It is possible to exert forces on small objects in a controlled fashion using a laser. Consider a laser beam incident upon a small glass prism as shown. The prism has an index of refraction $n = 1.6$ and the laser light is of frequency f. The prism is isosceles with a base of $2h$, a height w and a base angle of α.



(a) Sketch the possible paths for a ray of laser light entering the top half of the prism.

(b) Find the range of values for the angle α so that the light will exit from the long side of the prism as shown and find an expression for the deflection angle θ (you might want to do this first!). Only an approximate numerical value for α is required. Henceforth, you may assume that the angle α satisfies this condition.



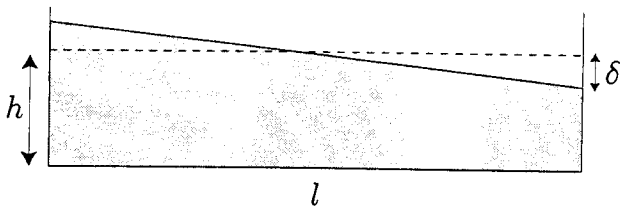
(c) The momentum p of a photon is related to its energy E by $E = pc$ where c is the speed of light. Find the vertical force exerted on the prism by a stream of r photons per second incident upon its top half. You may express your answer in terms of the deflection angle θ.

Question 3

In some large lakes, typically ones that are long and narrow, a strange phenomenon occurs. The water will oscillate up and down slowly first being high at one end and low at the other and then later reversing itself. It is much like the water in a bathtub sloshing back and forth and is not to be confused with regular waves on the water's surface. The phenomenon is called "seiching".

(a) How can seiching be initiated and how might it decay?

(a) Make a physical model for seiching and find an expression for a lake's period of oscillation. Possible factors include the lake's length l, width w, depth h, and amplitude of oscillation δ. Most simple models might begin by considering the lake to be a rectangular tank of water with the following geometry.



An exact solution is not expected for this question. Rather, a simpler system that you can imagine and that has characteristics in common with this problem can be used to model seiching and to estimate a period of oscillation.

You may compare your model with data from Lake Geneva in Switzerland and Lake Vättern in Sweden. Lake Geneva has a length of 60 km and an average depth of 150 m. Its oscillation period is 76 minutes with amplitudes of up to 2 m. Seiching on Lake Geneva can last for up to a week. Lake Vättern is 123 km long, has an average depth of 50 m and seiches with a period of about 3 hours.

Link to Solutions