Canadian Association of Physicists
1995 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee.

The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to, say, part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and part (d) of each question is very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6380$ km
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Magnetic constant	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	340 m/s
Boltzman constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273$°C

Part A: Multiple Choice

Question 1

A salvage ship tries to raise a sunken miniature submarine from the bottom of Lake Superior. The submarine and its contents have a mass of 72,000 kg and a volume of 18.9 m^3. What upwards force must be applied to raise the submarine? The density of water is 1000 kg/m^3.

(a) 1.8×10^5 N
(b) 2.0×10^5 N
(c) 4.8×10^5 N
(d) 5.2×10^5 N

Question 2

A little girl is playing with a toy pendulum while riding in an elevator. Being an astute and educated young lass, she notes that the period of the pendulum is $T = 0.5$ s. Suddenly the cables supporting the elevator break and all of the brakes and safety features fail simultaneously. The elevator plunges into free fall. The young girl is astonished to discover that the pendulum has,

(a) continued oscillating with a period of 0.5 s.
(b) stopped oscillating entirely.
(c) decreased its rate of oscillation to have a greater period.
(d) increased its rate of oscillation to have a lesser period.

Question 3

Two identical spring loaded, toy guns shoot projectiles straight upwards. The projectile in gun B has twice the mass as that in gun A. The projectile launched by gun A reaches a height H. Ignoring air resistance, the projectile launched by gun B would reach a height of,

(a) $H/4$
(b) $H/2$
(c) $H/\sqrt{2}$
(d) H

Question 4

An athlete of mass 75 kg runs at 10 m/s. What is his kinetic energy?

(a) 3750 J
(b) 7500 J
(c) 375 J
(d) 750 J

Question 5

The moon is about 60 earth radii from the centre of the earth and completes one orbit in about 28 days. Assuming that the moon executes a circular orbit about the centre of the earth, the acceleration of the moon (in units of g, the acceleration due to gravity at the earth's surface) is closest to,

(a) $g/30$
(b) $g/60$
(c) $g/3600$
(d) g

Question 6

A survey crew sights the top of a radio tower with a small telescope. The angle that the telescope makes with the horizontal is measured to be 53°. The crew moves the telescope 25 m closer to the base of the tower and repeats the process. The new angle that the telescope makes with the horizontal is 60°. What is the height of the tower assuming that the base of the tower is on the same level as the areas where the telescope measurements were made?

(a) 33 m
(b) 43 m
(c) 142 m
(d) 286 m

Question 7

Which of the following is **not** true for electromagnetic waves;

(a) they consist of oscillating electric and magnetic fields.
(b) they travel at different speeds in air depending on their frequency.
(c) they transport energy.
(d) they have no momentum.

Question 8

Two long parallel wires are placed side-by-side on a level table. The two wires carry equal currents in the same direction. Which of the following is true of the magnetic forces exerted on each wire by the other?

(a) One wire experiences a force upwards while the other experiences a downwards force.
(b) Both wires experience an upwards force.
(c) The two wires are attracted to one another.
(d) The two wires repel one another.

Question 9

A piano tuner uses a tuning fork which oscillates with a frequency of 440 Hz (middle A). Middle A on a piano is out of tune, producing a note of frequency f. The piano note and the tuning fork are heard to beat with a frequency of 3 Hz. When the tension in the piano string is decreased, the beat frequency also decreases. The out-of-tune frequency f must have been,

(a) 446 Hz
(b) 443 Hz
(c) 437 Hz
(d) 434 Hz

Question 10

Sudbury's Creighton mine is one of the deepest in the world (2.07 km). In this mine the conditions as compared to those at the surface are,

(a) lower air pressure, higher acceleration due to gravity.
(b) higher air pressure, lower acceleration due to gravity.
(c) higher air pressure, higher acceleration due to gravity.
(d) lower air pressure, lower acceleration due to gravity.

Question 11

A parallel network of resistors consists of a 1 Ω resistor placed in parallel with a 2 Ω resistor. This combination is, in turn, placed in parallel with a 4 Ω resistor. Further 8 Ω, 16 Ω, 32 Ω ... resistors are placed in parallel with the network one by one. To what value does equivalent resistance of the network converge upon?

(a) 0.5 Ω
(b) 2.0 Ω
(c) 0.0 Ω
(d) Does not converge to a finite value.

Question 12

A simple pendulum has a bob of mass 2 kg hanging on a cord of length 1 m. Suppose the pendulum is raised until it is horizontal (an angular displacement of 90°) and then released. What is the speed of the bob at the bottom of its swing?

(a) 9.91 m/s
(b) 19.6 m/s
(c) 3.13 m/s
(d) 4.43 m/s

Question 13

An organ pipe of length L is open at one end and closed at the other. What are the wavelengths of the three lowest frequencies that can be produced with this pipe?

(a) $4L,\ 2L, L$
(b) $2L,\ L,\ L/2$
(c) $2L,\ L,\ 2L/3$
(d) $4L,\ 4L/3,\ 4L/5$

Question 14

A satellite is in orbit at an altitude of one earth radius. What is the orbital speed of this satellite?

(a) 1.56×10^7 m/s
(b) 3.95×10^3 m/s
(c) 7.91×10^3 m/s
(d) 5.59×10^3 m/s

Question 15

A steady non-turbulent stream of water comes out of a tap and falls vertically downward. As it does so, the diameter of the stream appears to get smaller. What is the primary reason?

(a) The water's surface tension constricts the stream.
(b) Air pressure, which decreases with altitude, squeezes the stream.
(c) The water is accelerating under gravity and so the stream must get thinner as the flow rate (velocity times cross-sectional area) must be constant.
(d) The flow does not constrict; it is an optical illusion.

Question 16

A roller coaster car is on a track that forms a circular loop, of radius R, in the vertical plane. If the car is to maintain contact with the track at the top of the loop (generally considered to be a good thing), what is the minimum speed that the car must have at the bottom of the loop. Ignore air resistance and rolling friction.

(a) $\sqrt{2gR}$
(b) $\sqrt{3gR}$
(c) $\sqrt{4gR}$
(d) $\sqrt{5gR}$

Question 17

An object is placed 30 cm in front of a thin, spherical lens. An upright image, twice the size of the object, is formed by the lens. Which of the following attributes best describes the lens.

(a) diverging, focal length 20 cm
(b) converging, focal length 20 cm
(c) diverging, focal length 60 cm
(d) converging, focal length 60 cm

Question 18

A point charge $+Q$ is placed at the centroid of an equilateral triangle. When a second charge $+Q$ is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 4 N. What is the magnitude of the net force on the central charge when a third charge $+Q$ is placed at another vertex of the triangle?

(a) zero
(b) 4 N
(c) $4\sqrt{2}$ N
(d) 8 N

Question 19

Greenhouse "warming" is caused by an atmosphere which is ,

(a) transparent to visible light but opaque to I.R. light.
(b) transparent to both visible and I.R. light.
(c) opaque to both visible and I.R. light.
(d) warming due to the hot air from the world's increasing population.

Question 20

A particle of mass m and charge q is accelerated through a potential difference V to a velocity $\vec{v}$ towards the south. The particle enters a region with both a magnetic field $\vec{B}$ (pointing eastwards) and electric field $\vec{E}$ (pointing down). The particle travels at constant velocity through this region. The potential difference V must satisfy,

(a) $V = mE^2/2qB^2$
(b) $V = 2mE/qB$
(c) $V = E/qB$
(d) It is not possible for the particle to be undeflected by these fields.

Question 21

A space craft orbiting the moon with an orbital radius of 10^6 m has an orbital period of 45 minutes. When it drops down to a lower orbit of radius 10^5 m, its period has become,

(a) 36 minutes
(b) 85 seconds
(c) 54 minutes
(d) Need to know the mass of the moon for this problem.

Question 22

A simple pendulum is made from a 2 kg block of wood suspended from a light cord of length 1 m. When the pendulum is hanging in such a way that it is stationary and vertical, a bullet is shot horizontally into the block of wood where it sticks. The bullet has a mass of 10^{-2} kg and has a speed of 500 m/s just before its head-on collision with the block. The pendulum will swing to a maximum angle with the vertical of,

(a) 71.3°
(b) 23.4°
(c) 35.7°
(d) 46.8°

Question 23

Electronic components are tested for durability by subjecting them to high accelerations. This is achieved by placing them on a platform attached to a vertically hanging spring. The platform (and the component being tested) oscillate up and down with a frequency of 10 Hz when the spring is stretched by 10 cm and released. What is the maximum acceleration experienced by the electronic component?

(a) 10.0 m/s^2
(b) 2.00 m/s^2
(c) 395 m/s^2
(d) 158 m/s^2

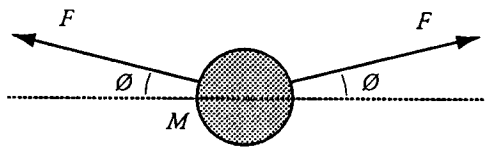
Question 24

A real battery can be considered to be an ideal battery of potential ϵ in series with an internal resistance r. Suppose that the battery is connected to a single "load" resistance R. For what value of R will the power dissipated by the load be a maximum?

(a) r
(b) $r/2$
(c) $2r$
(d) $10r$

Question 25

Two trucks pull on either end of a rope, each with a force of 10,000 N. In the centre of the rope, a ball of mass 20 kg is hanging.



The angle θ that the rope makes with the horizontal is,

(a) 56°
(b) 5.6°
(c) 0.56°
(d) 0°; the rope will obviously be horizontal in this situation.

Part B

Question 1

Physicists at UBC do microgravity experiments on board a NASA KC-135 (Boeing 707). The KC-135 flies parabolic flight paths to follow as closely as possible the trajectory and speed of a free-falling body. Once the plane is on the parabolic flight path (both ascending and descending), its engines are only used to overcome air resistance. The aircraft is known, with some justification, as the "Vomit Comet". The KC-135 cannot fly at a speed of less than 300 km/hr (as it has to be under control at all times) or more than 500 km/hr (or the g-forces in recovery will be too great).

(a) While the plane is executing its parabolic trajectory at an altitude of 10,000 m, a physicist of mass $m = 65$ kg demonstrates her "weightlessness" by floating freely in the large open cargo bay. Draw a force diagram for the physicist showing all forces acting on her body. Find the force of gravity acting on the physicist and find her net acceleration. Explain in your own words, and referring to your preceding calculations, why people call this experience "weightlessness".

(b) At what angle to the horizontal should the KC-135 enter its parabolic path?

(c) How much time do the physicists get under "weightless" conditions?

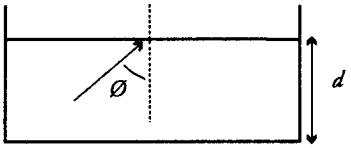
(d) While an aircraft is in level flight and travelling at constant velocity, passengers on board are said to experience a 1-g "force" downwards and can walk around the cabin normally. Sketch the trajectory of the path the aircraft would have to follow so that it could execute a 1-g vertical loop. In this situation, the aircraft must execute a full vertical loop while at all times allowing the passengers to experience only a 1-g "force" towards the floor of the cabin. All on board would be able to walk around normally at all times. What is the acceleration of the aircraft, in units of g, when it is at the top, and bottom of the loop? What is the acceleration of the aircraft, in units of g, when it is headed vertically upwards and downwards?

Question 2

A tank of salt water is allowed to stand undisturbed for a long time. The tank has a square footprint and measures 3 m by 3 m. The depth of the water in the tank is $d = 1$ m. In the absence of any agitation, the concentration of salt in the water is not uniform but increases with depth. Since salt water is more dense that pure water, the density of the water also increases with depth and hence, the index of refraction of the water also increases with depth. Assume that the index of refraction of the water at its surface is $n_o = 1.3$ and the index increases linearly and at the rate of

$\alpha = 0.05\ \mathrm{m}^{-1}$. The walls and floor of the tank are made of clear glass and the air surrounding the tank has an index of refraction $n_a = 1$. This tank can be used to model the optical properties of a variety of situations where the index of refraction varies with altitude.

(a) Suppose, for parts (a) and (b), that we make the simplification that the index of refraction of the water is uniform and has the value $n_o = 1.3$ throughout the tank. A laser beam is shone through the floor of the tank towards the surface of the water as shown. When the beam is pointing straight up, it passes through the salt water and exits into the air above. As the angle between the beam and the normal to the surface is increased, a point is reached where the beam no longer exits the water but is internally reflected back into the tank. Calculate the angle at which this total internal reflection occurs.



(b) A small light bulb is placed directly under the middle of the tank. Looking down from above at the surface of the water, what area on the surface appears to be illuminated by the light? Note that light from a light bulb travels in all directions.

(c) Now let us consider the more complicated case where the water is not of uniform density. Treat the salt water as a large number of layers, each with a distinct index of refraction and sketch the path of the laser beam in part (a). At what angle, from the normal to the bottom of the tank, did the laser beam enter the water so that it would be internally reflected at the top surface? Sketch the path of a ray which entered the tank at an angle greater than this.

(d) Now consider air above a hot roadway. At the surface of the roadway, the temperature of the air is $T_h = 60°\mathrm{C}$ whereas at a height of $l = 1$ m and above, the air is a cool $T_c = 30°\mathrm{C}$. The index of refraction $n(T)$ of the air as a function of temperature is related to the air's density $\rho(T)$ according to the relation,

$$n(T) - 1 \propto \rho(T)\ ,$$

where T is the absolute temperature of the air in degrees Kelvin. You may assume that the density of the air is, in turn, inversely proportional to the air's temperature. The index of refraction of air at 15°C and atmospheric pressure is 1.000276. Sitting in a car with your eyes 1.5 m above the roadway, how far is it in the distance that the roadway appears to shimmer? This is, of course, a mirage.

Question 3

Estimate from your basic bodily dimensions your natural walking speed. Explain clearly, in words and diagrams, the physics you use to make this calculation. If time allows, go beyond your first basic calculation to discuss refinements and improvements to your model for walking. Be creative.

Link to Solutions